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UNIVERSITÀ DEGLI STUDI DI MILANO
DIPARTIMENTO DI INFORMATICA

CONNETS LAB

NETWORK EVOLUTION

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Based on the PhD thesis of Alessia Galdeman and Manuel Dileo

CONNETS LAB

DIPARTIMENTO DI INFORMATICA

NETWORK SCIENCE

Sabrina Gaito
Head of the Lab

Multilayer Networks

Network Evolution

Temporal Networks

Cheick Ba

Alessia Galdeman

ARTIFICIAL INTELLIGENCE

Matteo Zignani

GNN

Manuel Dileo

LLM

Alessandro Cagiano

COMPUTER NETWORKS

Christian Quadri

Edge computing

Autonoumous driving

Alberto Bertoncini

NETWORK EVOLUTION

Every network exhibits a distinct evolution signature that shapes its peculiar structural and dynamical characteristics.

How to identify each network's distinctive evolutionary signature?

By defining and constructing a network evolution profile

a concise yet comprehensive and explainable quantitative descriptor

Able to capture the mechanisms underlying its evolution and temporal dynamics

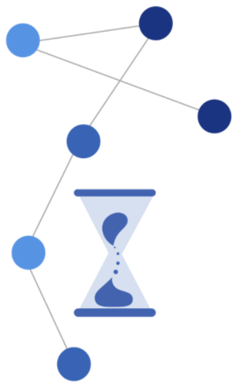
Connets LAB
Computer Science
UNIVERSITÀ DEGLI STUDI DI MILANO

The **CONTEXT**

- Past research mainly focused on static networks: developing theoretical frameworks, scalable algorithms, and deep knowledge of network structures;

- In recent years researchers recognized the complexity and time-varying nature of large systems;

We need time labels

TEMPORAL  NETWORKS

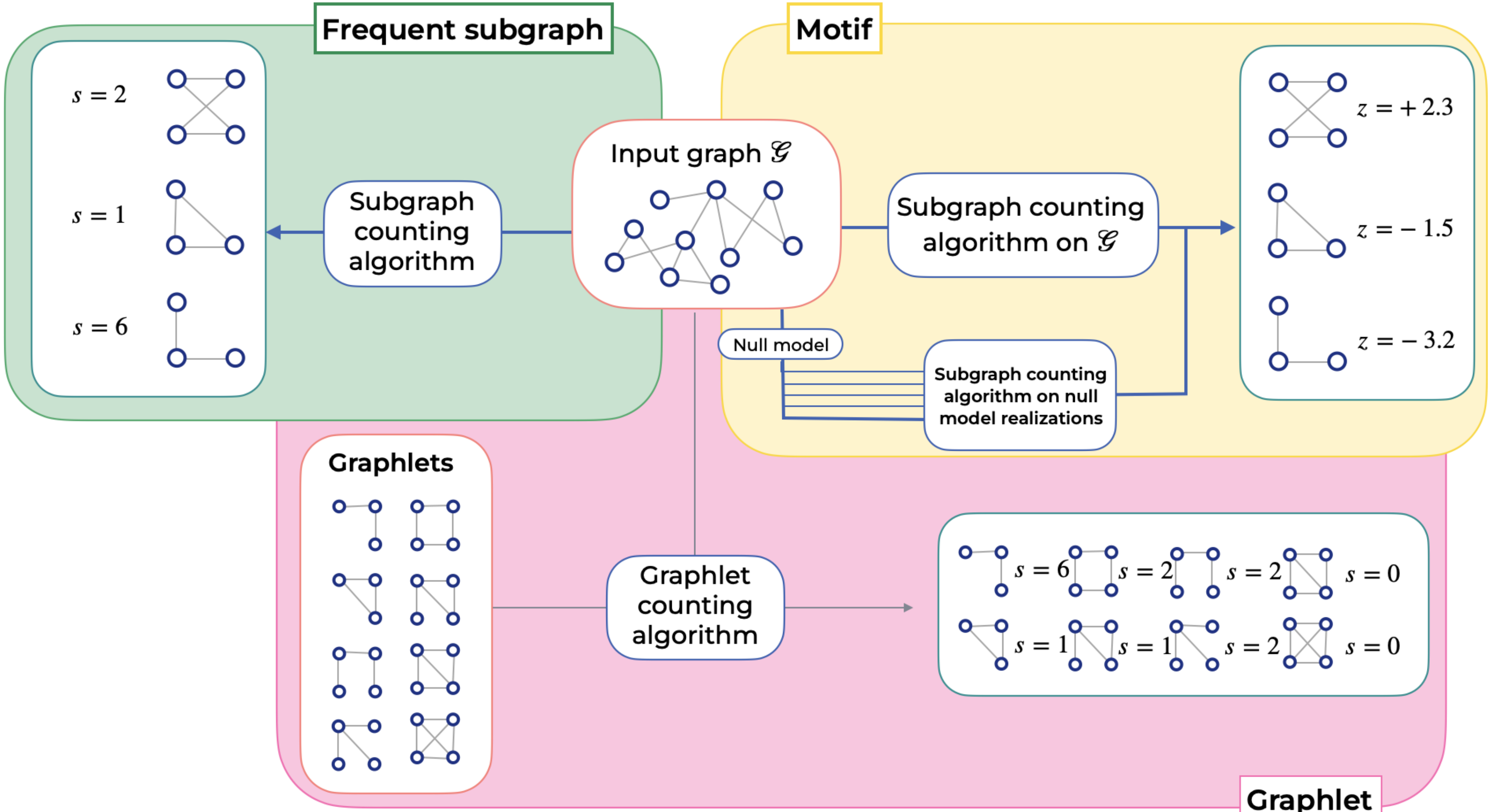
The **CONTEXT**

TEMPORAL NETWORKS   NETWORK EVOLUTION

When studying temporal networks, **we talk about** network evolution **if the focus is on the mesoscopic** mechanisms **that drives the** growth **of a network**

The **CONTEXT**

FREQUENT SUBGRAPH METHODS



Even if they are time-labelled

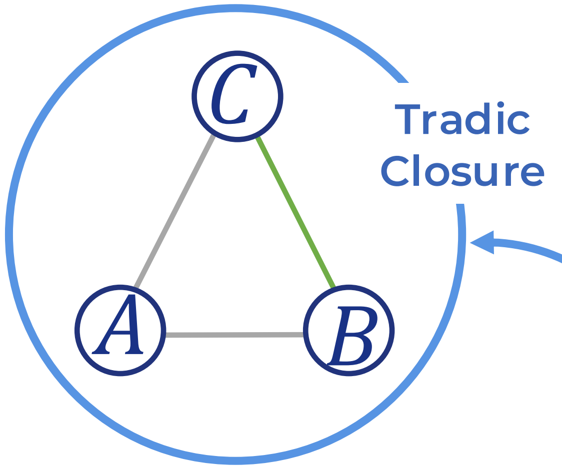
They do not really describe the evolution mechanisms

GRAPH EVOLUTION RULES

DEFINITIONS, FORMALISMS, MINING ALGORITHMS, AND VISUALIZATION

Graph evolution rules

REASONS WHY



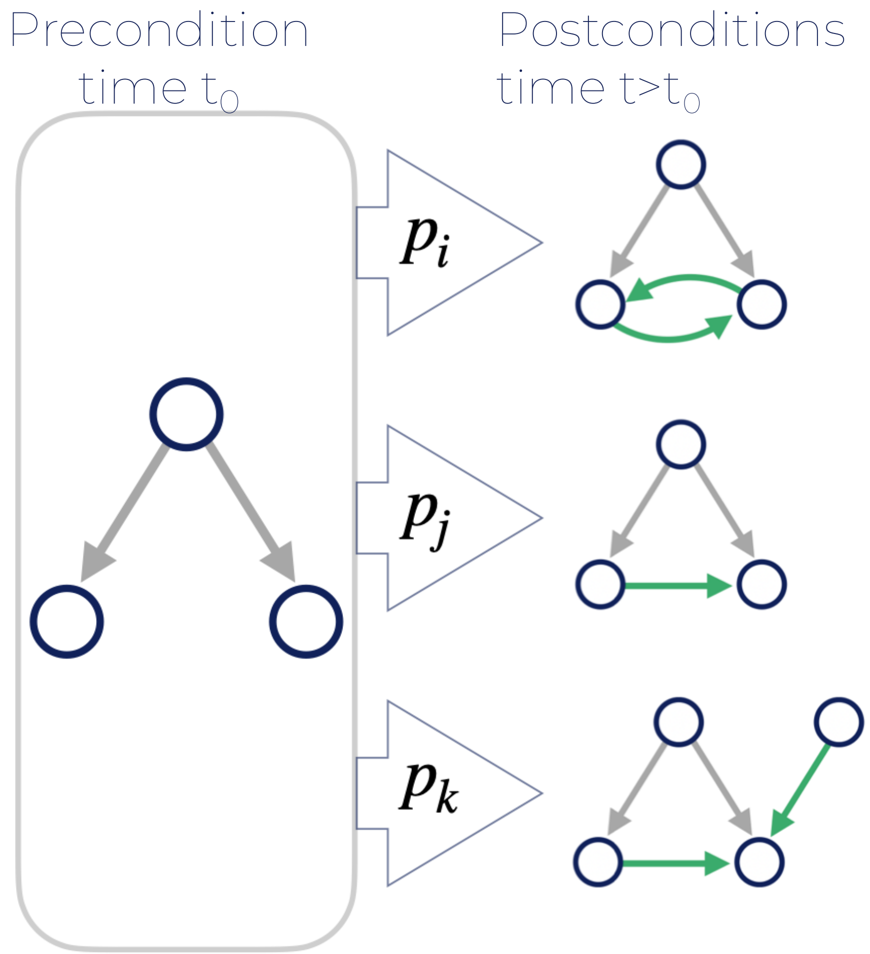
Several models, mechanisms and measures have been proposed to describe the network growth

BUT

- They assume that the growth is guided by a single parameterized mechanism
- Identifying which mechanism plays a more important role is challenging

Graph evolution rules mining can detect all evolutionary behaviors, while avoiding any a-priori mechanism

IDEA: Graph Evolution Rules



- Inspired by association rules in the data mining field, a GER consists of a precondition and a set of postconditions
- A subgraph matching the precondition evolve in the different postconditions with the associated probability

The **PITFALLS** of **GERs**

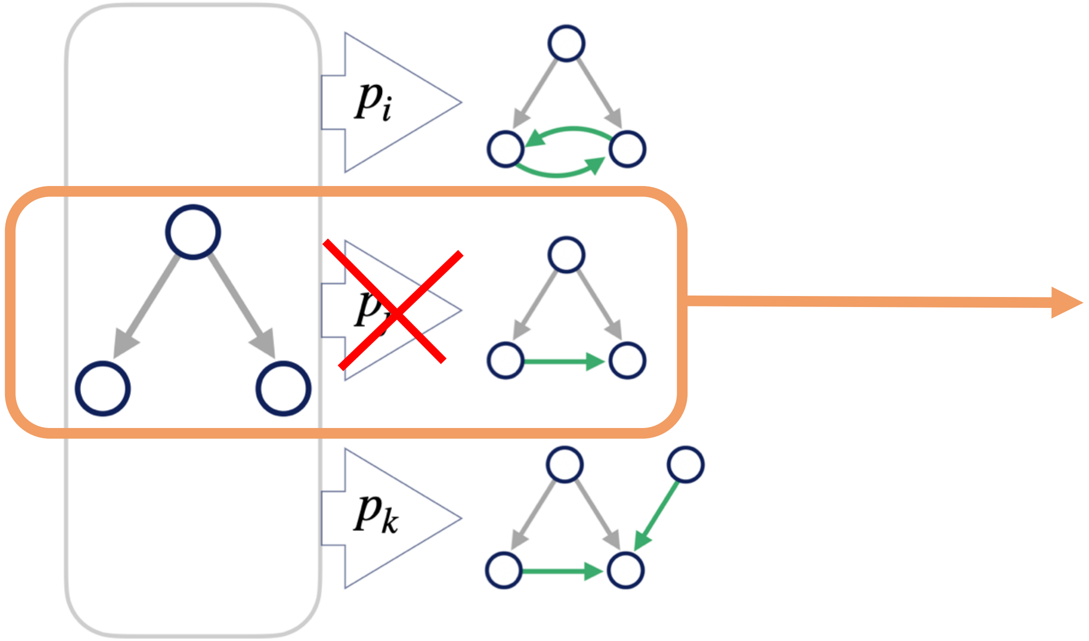
Very few works propose algorithm to find GERs

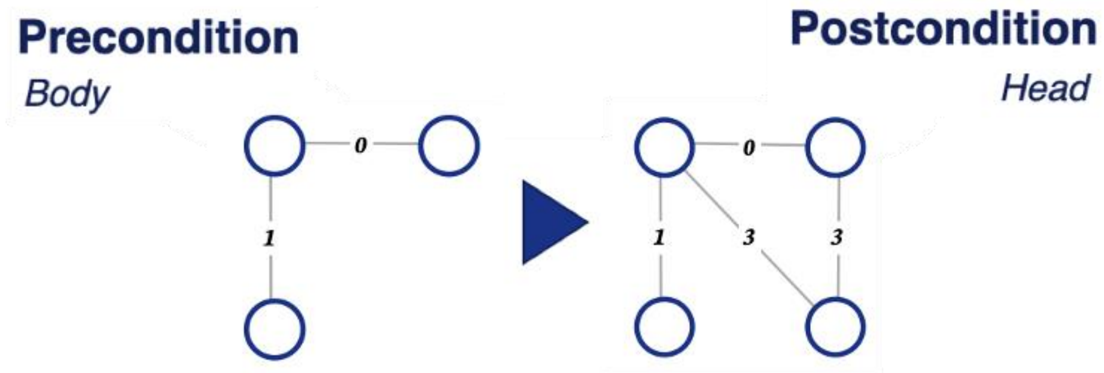
&

They all find stand-alone (independent) rules

GENERAL
GRAPH EVOLUTION RULES

STAND ALONE
RULES





The **PITFALLS** of **GERs**

Existing stand-alone algorithms lacks of:

CANONICAL FORM
so that results remain comparable
even after parallelization

Ad-hoc NULL MODEL
extract statistically significant rules

ANALYTICAL TOOL
to extract insights and reaching
interpretability

No algorithms for general graph evolution rules

Methodological FRAMEWORK for Network Evolution Profile

Main **CONTRIBUTIONS**

GERANIO FRAMEWORK

Modeling FRAMEWORK
Unified framework to model evolving networks

NULL model
Find statistical significant rules

TULIP algorithm
New algorithm to mine general GER

Evolutionary PROFILE
Tool for immediate analysis of evolutionary behavior

CANONICAL coding
Assignment of unique codes to isomorphic rules

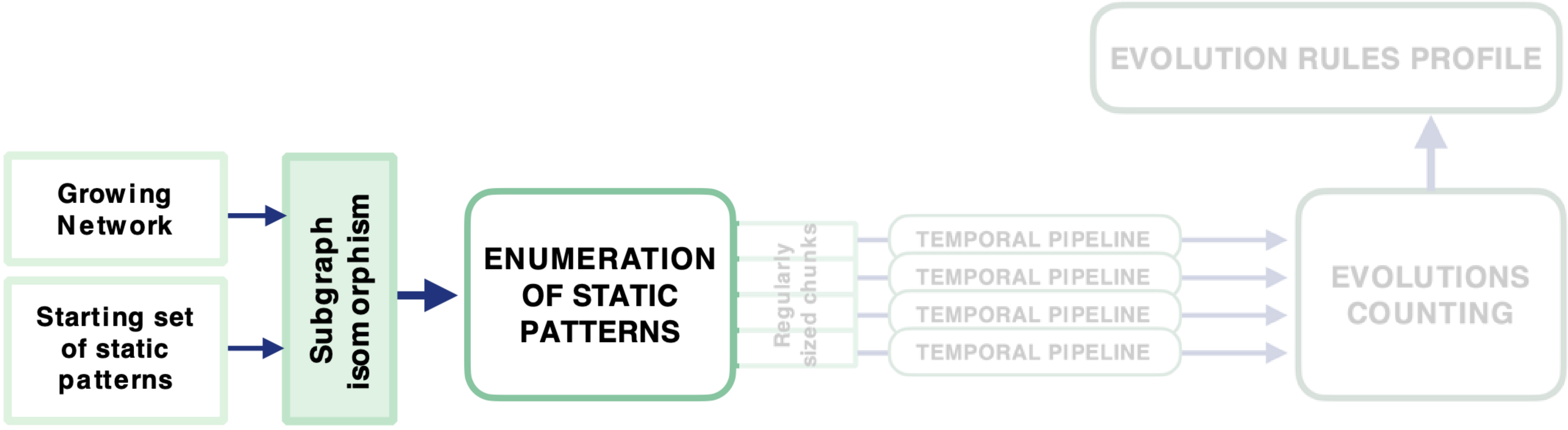
GERANIO

The **TULIP** ALGORITHM

TEMPORAL SUBGRAPHS FOR EVOLUTIONARY PROFILING

The **PIPELINE**

ENUMERATION OF PATTERNS

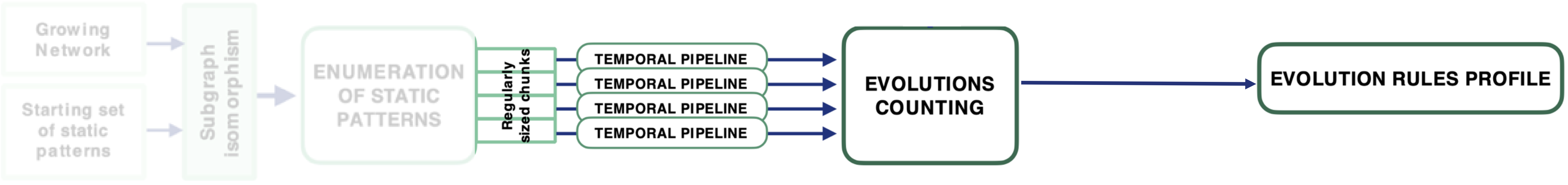


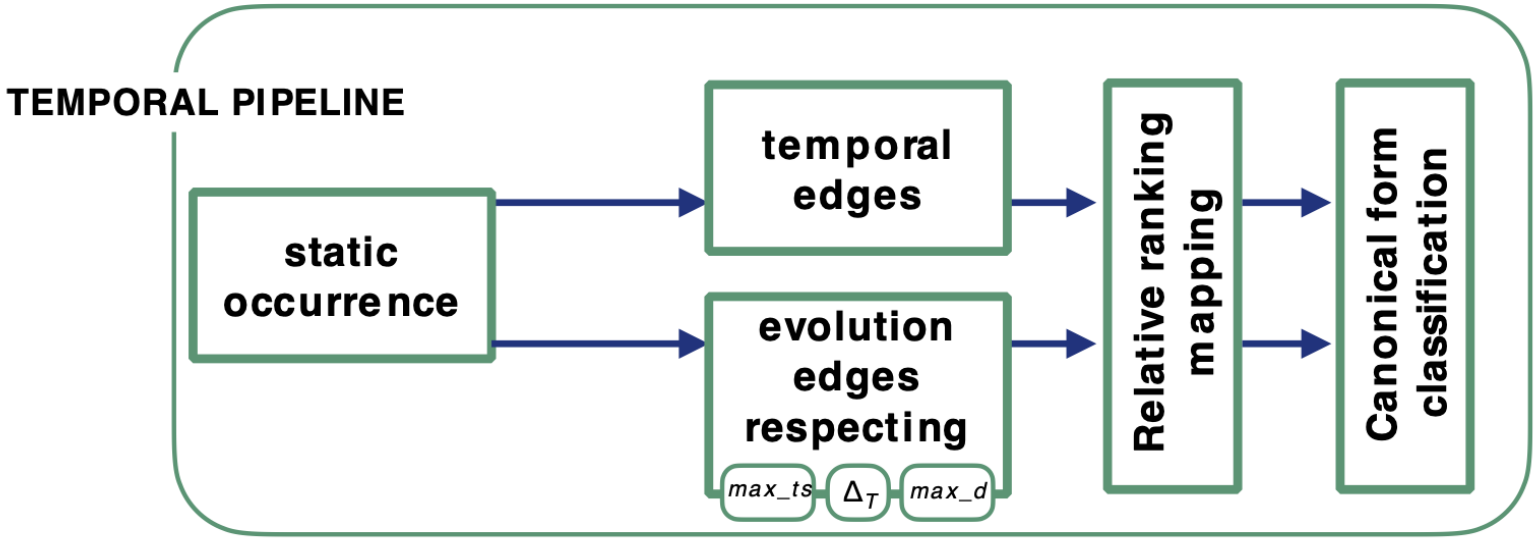
Graph mining is already a complex task, and temporal graph mining comes with additional constraints.

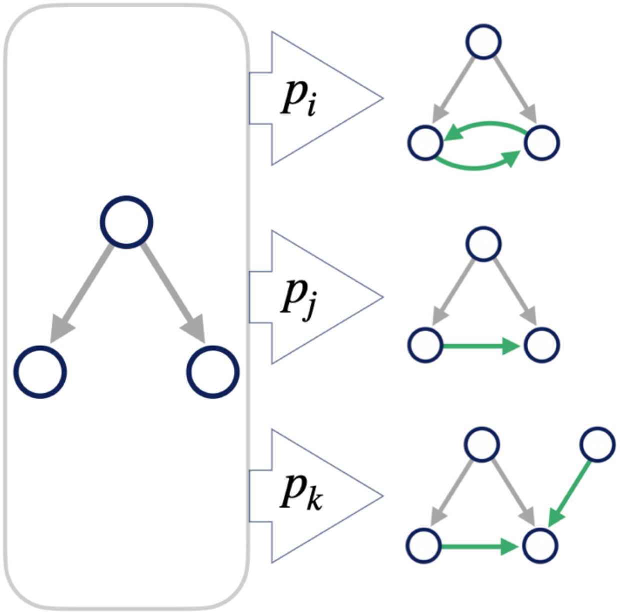
The idea for the tulip algorithm is to enumerate static patterns, and then extract the temporal information from edges that we know exists.

The **PIPELINE**

TEMPORAL PIPELINE



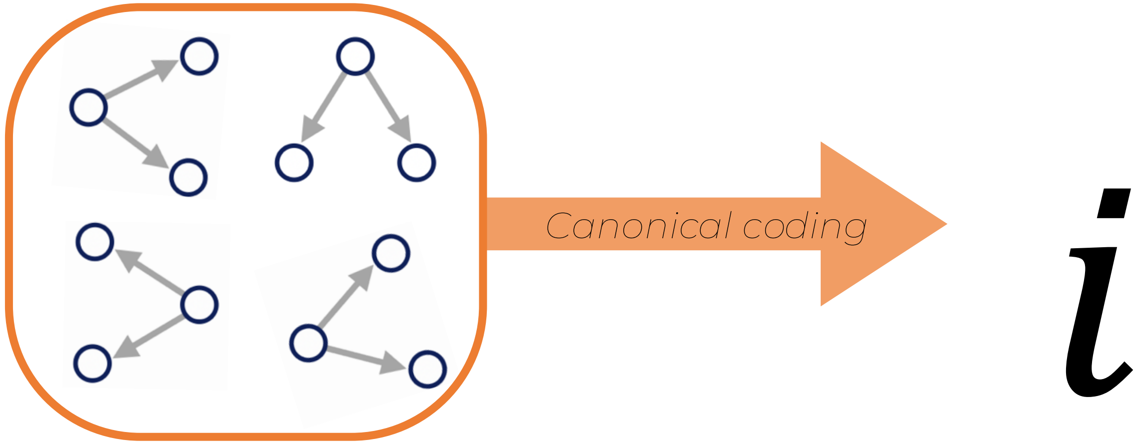




The temporal pipeline first extract the temporal information on enumerated edges and then search for the evolutions of the found temporal patterns

in parallel on chunks of enumeration, so results need to be canonically categorized

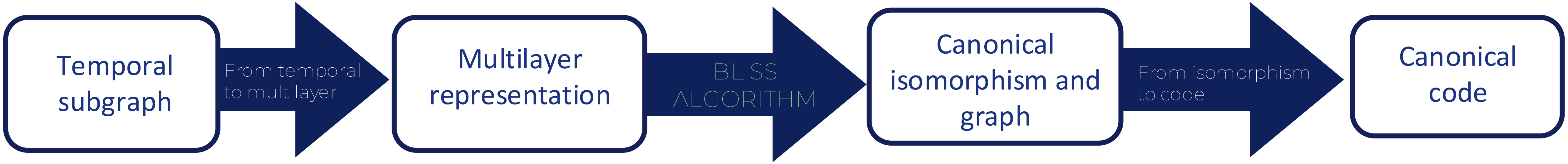
The CANONICAL CODING



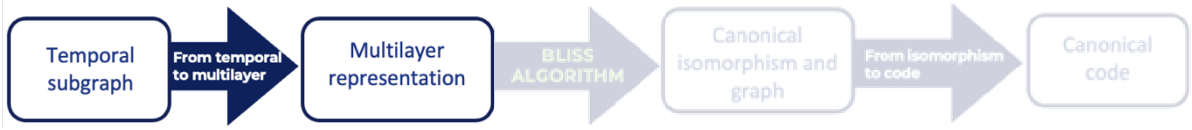
Obtain a unique code for all isomorphic subgraphs

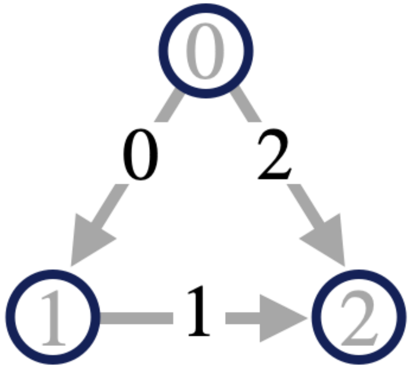
The **CANONICAL FORM**

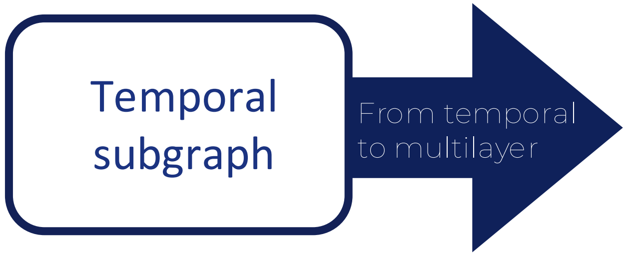
THE PIPELINE



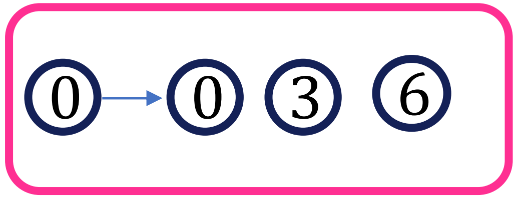
The **CANONICAL FORM**

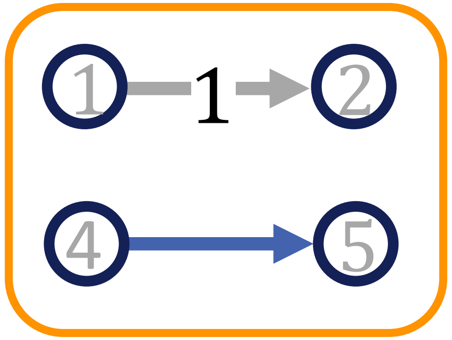


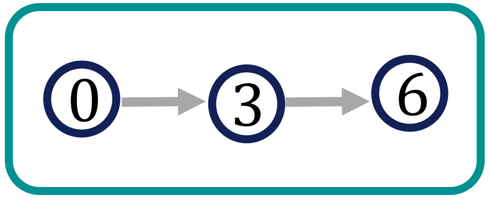


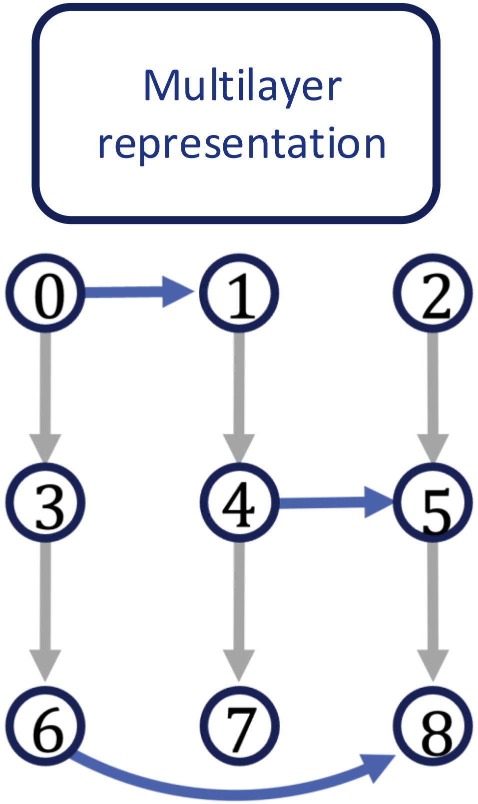


1. Duplicate nodes t times, where t = number of timestamps
2. Remap temporal edges in the correct “layer”
3. Insert pillar edges to ensure the temporal order of the multilayer representation

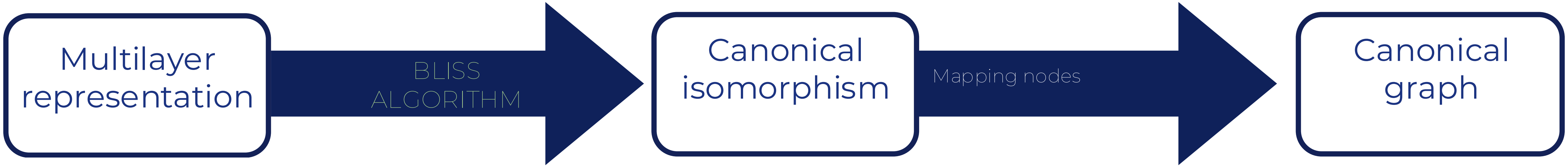


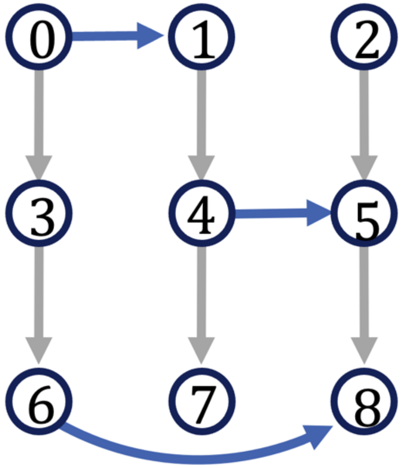






The **CANONICAL FORM**

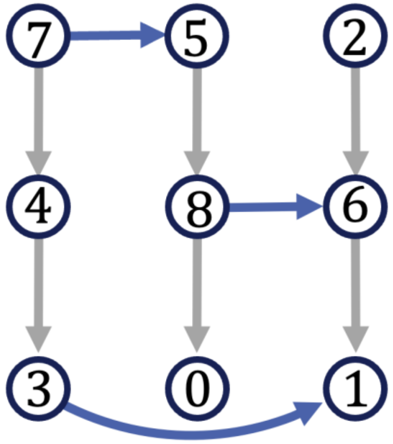




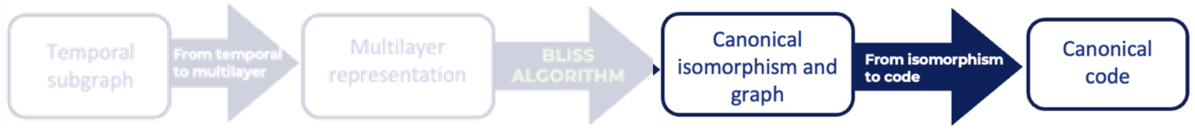
*The **canonical isomorphism** is a mapping of nodes to go from a graph to its canonical form (representative of the class of isomorphic graphs)*

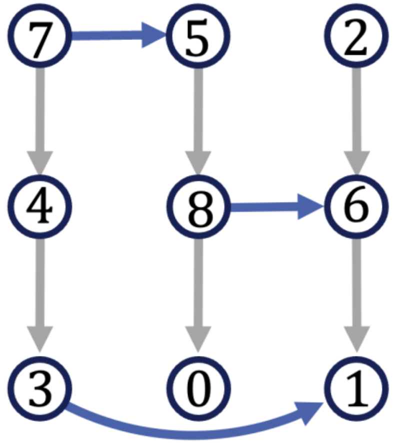
{0:7,
1: 5,
2:2,
3:4,
4:8,
5: 6,
6:3,
7:0,
8:1}

***Mapping nodes** means simply using the canonical isomorphism to relabel nodes, keeping the existing edges*



The **CANONICAL FORM**





1. Obtain the adjacency matrix of the canonical graph

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

2. Concatenate all lines and prepend a '1'

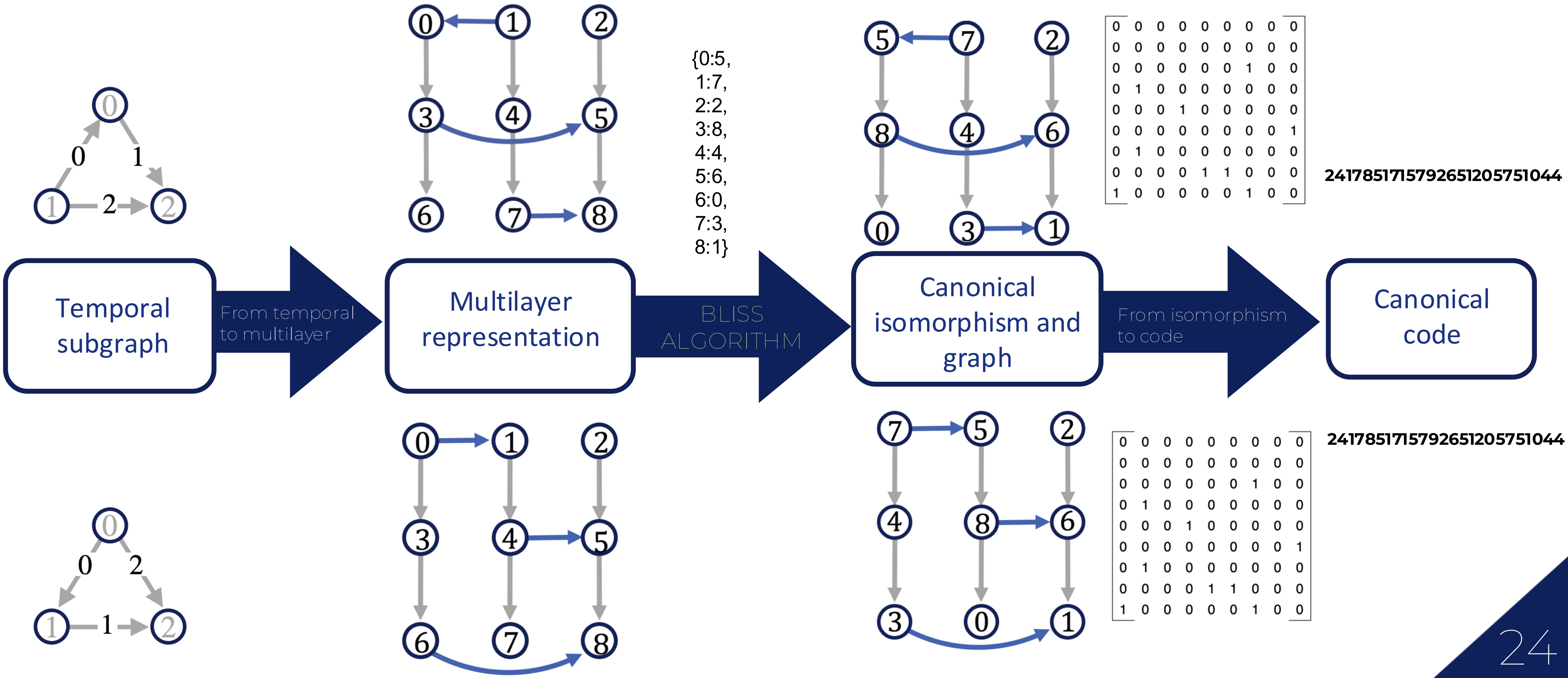
1000000000000000000000000100010000000 [...]

3. Transform the binary number into an integer

2417851715792651205751044

Canonical code

The CANONICAL FORM

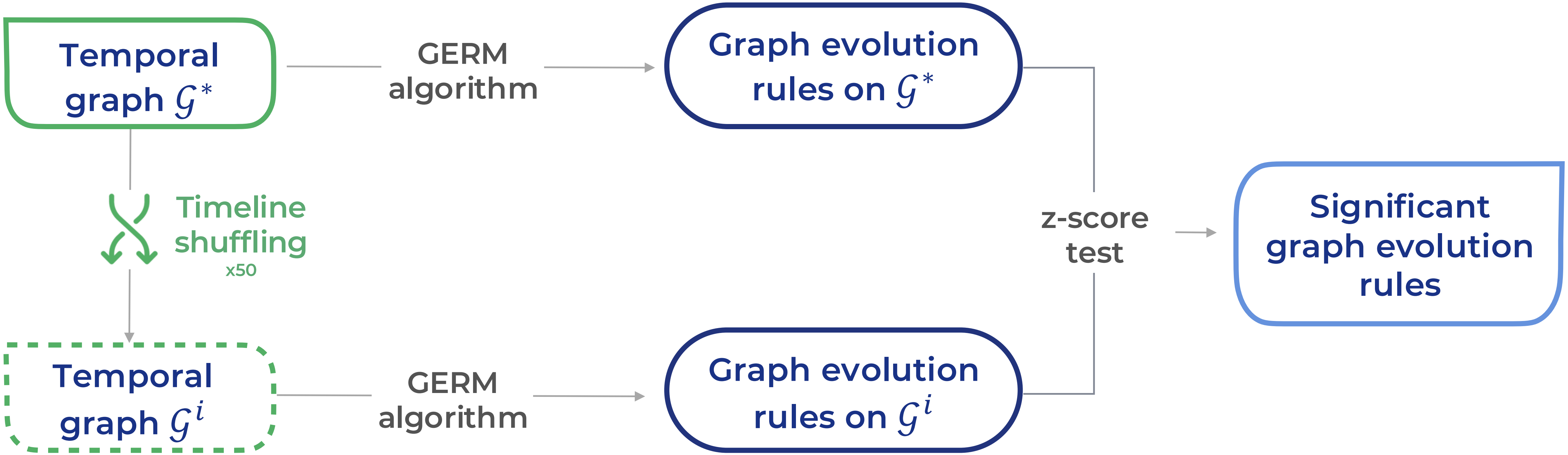


The NULL MODEL

WHY?

To evaluate the statistical significance of rules and therefore find GERs that characterize the evolution of a specific network

The **PIPELINE**



Galdeman et al. (2023), Unfolding temporal networks through statistically significant graph evolution rules.
Proceedings of the IEEE International Conference on Data Science and Advanced Analytics (DSAA 2023)

The EVOLUTIONARY PROFILE

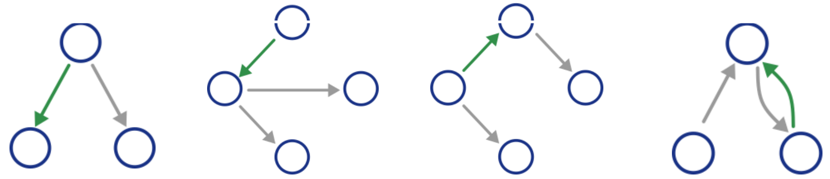
WHY?

- To compare networks evolutionary behavior easily
- It represents the final output of the framework

HOW?

Probability distribution over rules' frequency, each position refers to a specific rule

$$\left[\ 0.01 \quad 0.64 \quad 0.27 \quad 0.08\ \right]$$



The **EVOLUTIONARY PROFILE**

Every network has its own **evolution signature**

We capture it through an interpretable, quantitative fingerprint

the **network evolution profile**,

a **compact, complete, and explainable** representation

that encodes all the key mechanisms driving

the network's evolution and growth over time.

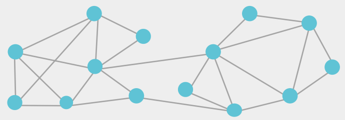
Some

APPLICATIONS

The **EVOLUTIONARY PROFILE**

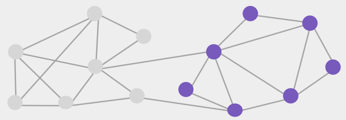
DIFFERENT LEVEL OF APPLICATION

GRAPH



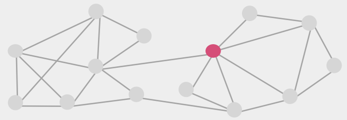
Evaluate evolutionary behavior for the whole graph

COMMUNITY



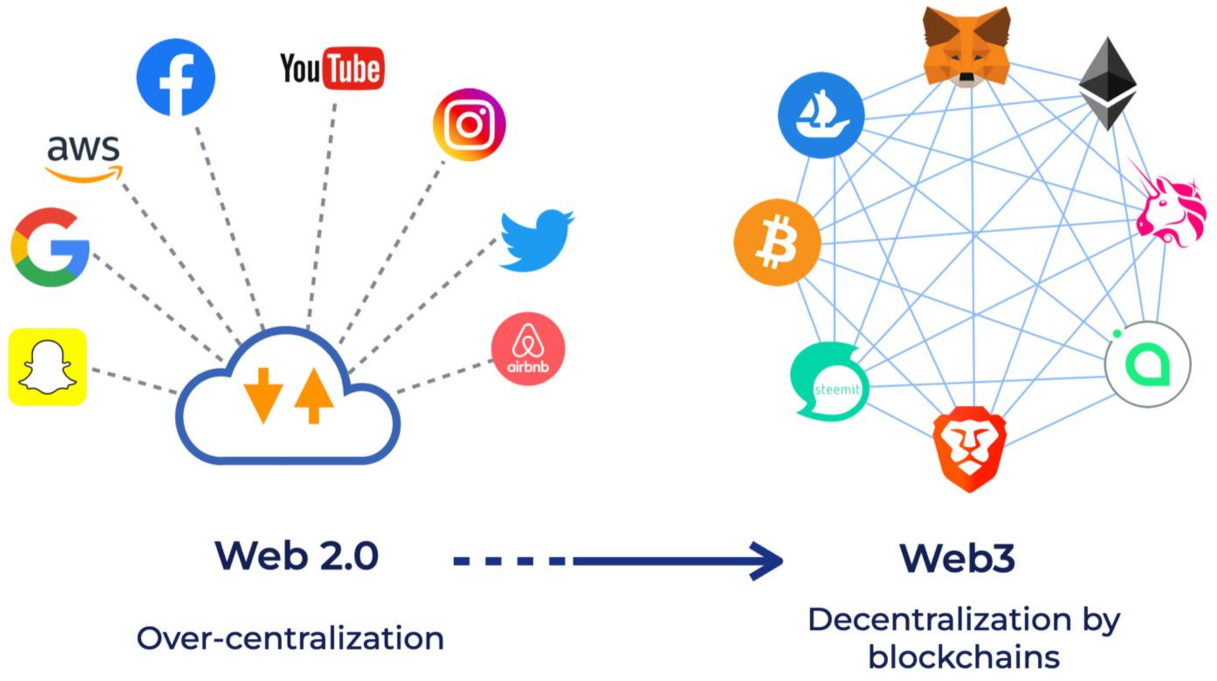
Consider the evolutionary behavior of each community

NODE



Compare the evolutionary behavior of each node’s ego-network

Temporal data: the WEB3



Web3 data

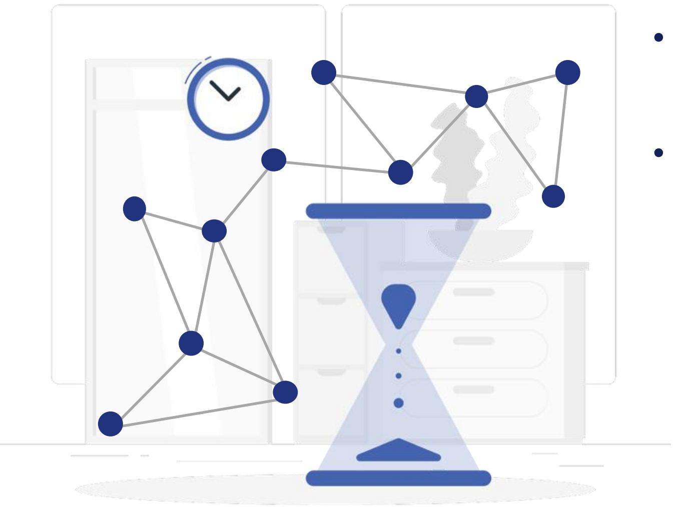
- Huge volume of high resolution data
- Available and affordable by API
- Timestamped and validated
- Heterogeneous interactions

Research Question

Are Web3 platforms really different from web2 networks?

Temporal networks

CHALLENGES AND DATA SOURCES



- An interesting but yet not fully explored field, mainly due to the lack of temporal data
- Thanks to the web3 development, we have enough data to develop solid temporal methodologies

WEB3 data

Blockchain-based online social netoworks

Social networks based on a reward-system for content creator and curators Examples: Steemit, Hive, and Galxe

Non-fungible tokens

Networks of NFT trades on different markets Examples: CRyptokitties, OpenSea, and Decentraland

Complementary currency

Exchange of a complementary currency through the blockchain technology. Examples: Sarafu, and Circle

Stable coins

UC-social

Messages on a social network at UC Irvine college

N 2k | E 20k

Messages

Stack Overflow

Answers-to-questions interactions on the Stack Exchange website

N 2M | E 13.5M

Comments

Enron

Email communication records among Enron employees

N 32k | E 107k

Email

Bitcoin Alpha

Expression of trust on the Bitcoin Alpha platform

N 3.6k | E 22.6k

Trust

Sarafu

Complementary currency exchange within the Sarafu project

N 40k | E 143k

Financial

Cryptokitties

NFT exchanges on the Cryptokitties marketplace

N 100k | E 725k

NFT

Opensea

NFT exchanges on the Opensea marketplace

N 214k | E 962k

NFT

Vote

Voting to posts or comment

N 758k
E 109M MultiE 590M

Transfer

Cryptocurrency exchange on the Blockchain-based social network Steemit

N 474k
E 4M MultiE 59.4M

steemit

Follow

Cryptocurrency exchange on the Blockchain-based social network Steemit

N 1.4M
E 117M MultiE 136M

Comment

Comments on Steemit to posts or other comments (Reddit like)

N 625k
E 20M MultiE 70M

Citation

Citation network from the Digital Bibliography & Library Project (DBLP)

N 12k | E 47.6k

dblp
computer science bibliography

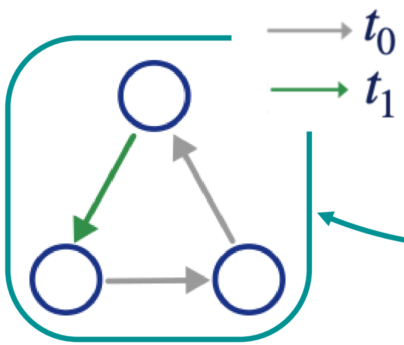
Collaboration

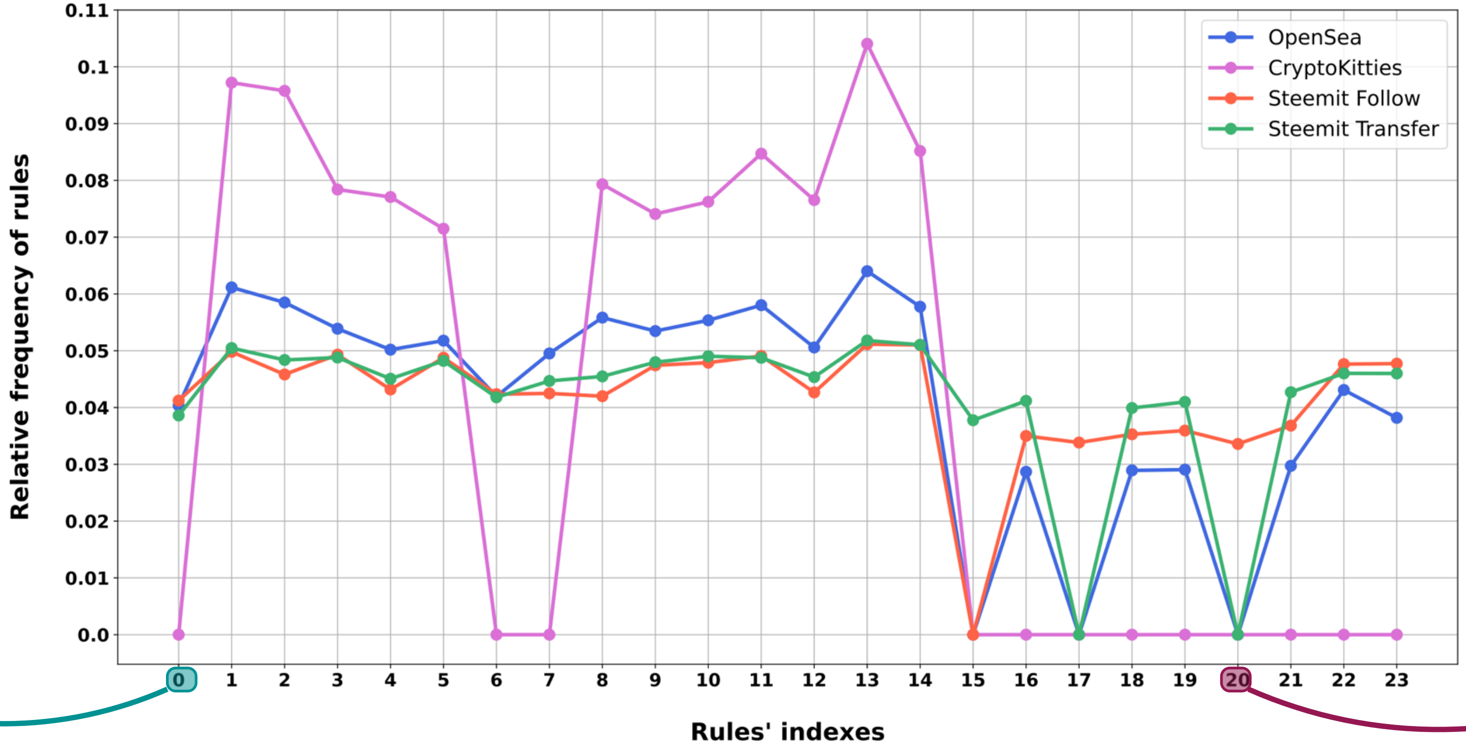
Co-authorship network from the Digital Bibliography & Library Project (DBLP)

N 129k | E 277k

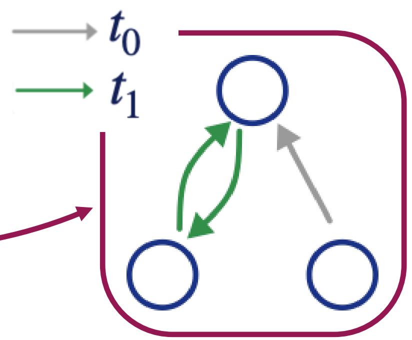
Graph **LEVEL**

Not in the frequent GER set for the cryptokitties market





Frequent only in Steemit follow (the only social network)



Both cases are explainable with the nature of the network itself

Galdeman et al. (2022), Disentangling the Growth of Blockchain-based Networks by Graph Evolution Rule Mining
. Proceedings of the IEEE International Conference on Data Science and Advanced Analytics (DSAA 2022)

Node **LEVEL**

CASE STUDY

We applied our approach to Sarafu, a complementary currency platform with rich temporal data. It represents a contemporary human complex system because it was used for humanitarian aid during COVID-19

412 050 BY **40 343**

Transactions *Users*

Jan20

Jun21

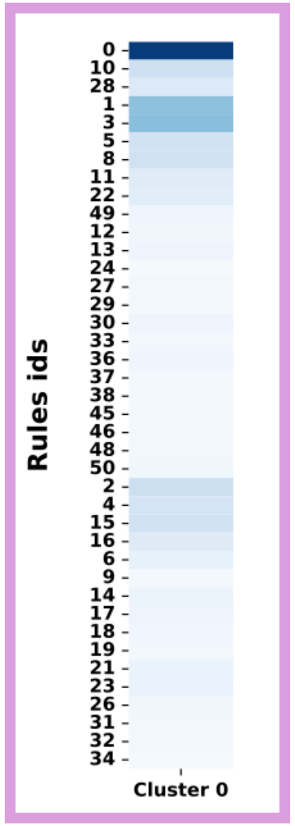
How do single nodes evolve in this humanitarian context?

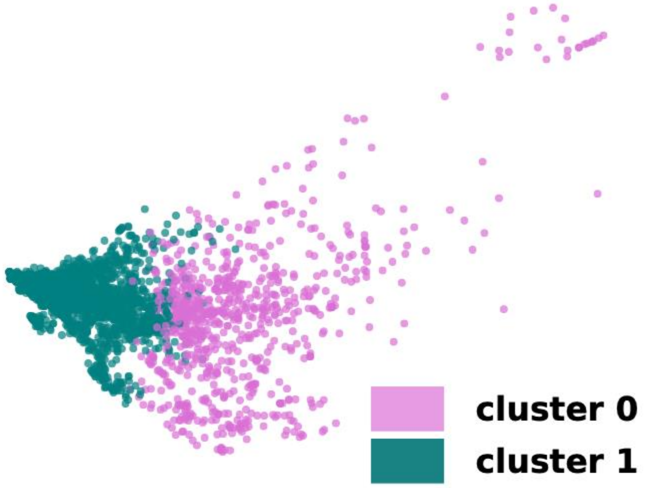
Node **LEVEL**

3 207

Ego Networks with consecutive timestamps and at least 116 edges

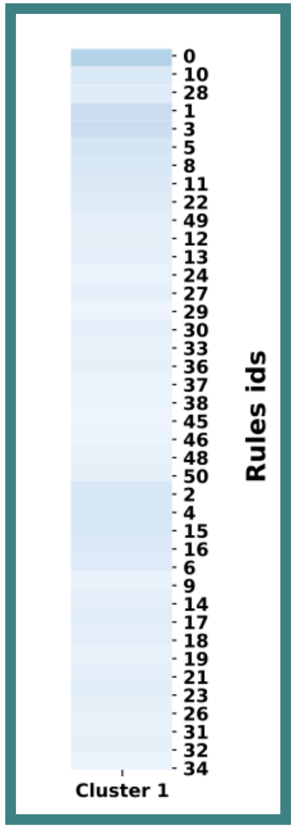
40 **distinct graph evolution rules found**





2 **distinct evolutionary behaviors**

- One group of users whose evolutionary behavior is dominated by single-link expansion
- Other group with homogeneous evolutionary behavior over expansion rules



Galdeman et al. (2024), Graph evolution rules for node temporal behavior representation
Proceedings of Discovery Science.

Community **LEVEL**

CASE STUDIES

Stack Overflow

Answers-to-questions interactions on the Stack Exchange website

Comments

Sarafu

Complementary currency exchange within the Sarafu project

Financial

DBLP

Citation network from the Digital Bibliography & Library Project (DBLP)

Citation

Enron

Email communication records among Enron employees

Email

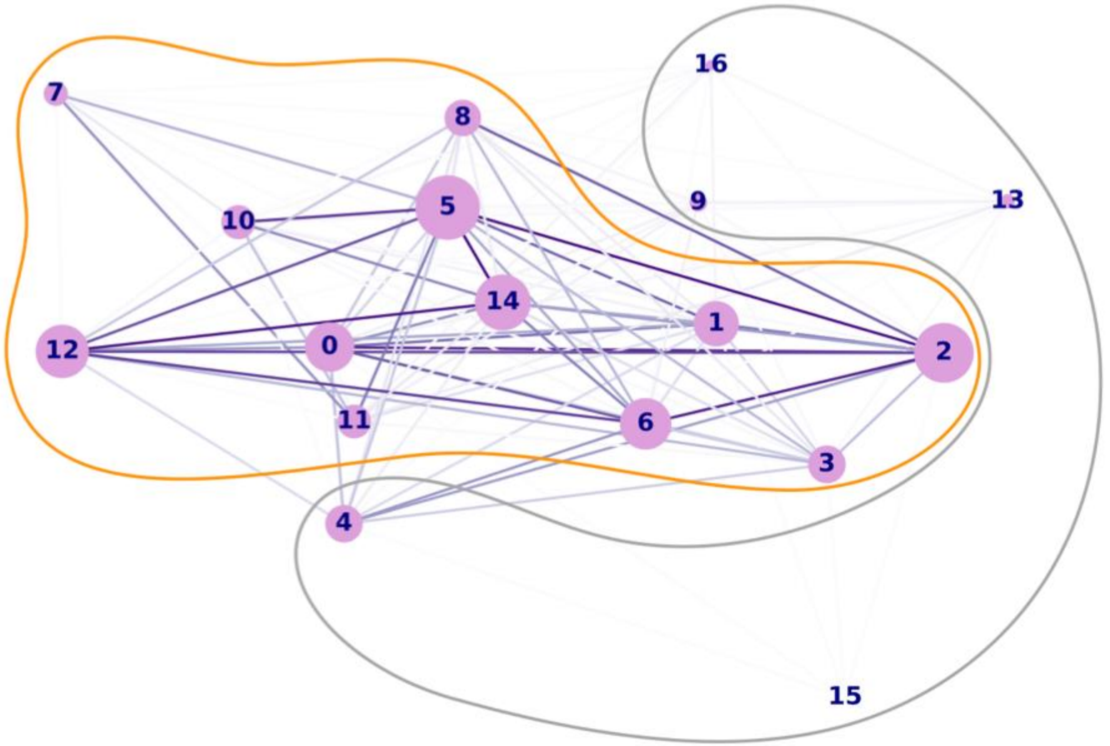
Do close communities also share similar evolutionary profile?

Galdeman et al. (2025), Graph evolution rules meet communities: assessing global and local patterns in the evolution of dynamic networks
In IEEE Big Data Mining and Analytics, 2025.

Community LEVEL

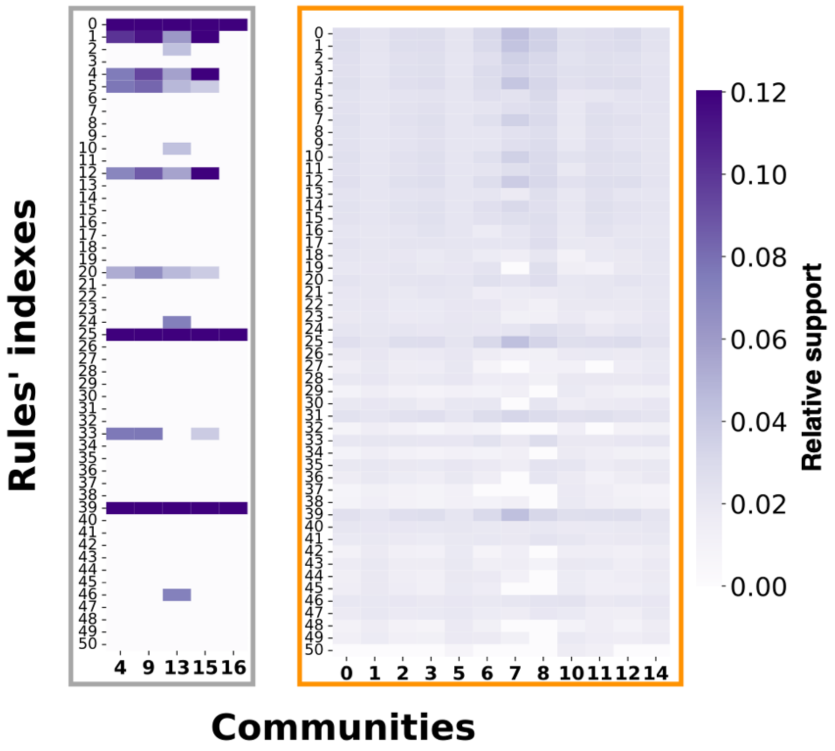
ENRON EXAMPLE

The community graph, where each node represents a community, so a group of nodes, and links between communities are weighted based on the number of edges between them in the original graph.



The position of the communities in the graph relates with the evolutionary profile

The evolutionary profiles represented as an heat map, each column is a evolutionary profile and rows are rules.



Two groups of profiles that correspond to the more central group of connected communities and the more isolated one.

Tulip **GER**

CASE STUDIES

UC-social

Messages on a social network at UC Irvine college

Messages

Sarafu

Complementary currency exchange within the Sarafu project

Financial

Steemit

Cryptocurrency exchange on the Blockchain-based social network Steemit

Financial

Cryptokitties

NFT exchanges on the Crptokitties marketplace

NFT

Bitcoin Alpha

Expression of trust on the Bitcoin Alpha platform

Trust

FUTURE RESEARCH:

NETWORK SCIENCE MEETS AI

IN GRAPH EVOLUTION RULES

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UNIVERSITÀ DEGLI STUDI DI MILANO

Machine learning general frameworks for temporal heterogeneous graphs: predictive methods, interpretability, applications, benchmarks

PhD Thesis: Manuel Dileo

Supervisor: Matteo Zignani

Co-supervisor: Sabrina Gaito

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UNIVERSITÀ DEGLI STUDI DI MILANO

ML on Temporal Heterogeneous Networks

CHALLENGES

Predictive methods: Lack of an unified framework to handle temporal, heterogeneous, and multimodal information on graphs.

Interpretability: Lack of proper baselines, which makes difficult to argue that we are making progress. Lack of explainability methods and benchmarks.

Datasets and applications: Lack of standard benchmarks, scarcity of datasets across diverse domains (mainly event networks).

DIFTHE: A discrete-time deep learning framework for temporal heterogeneous networks forecasting

Key idea: design the computation over temporal heterogenous graphs with **several modules**

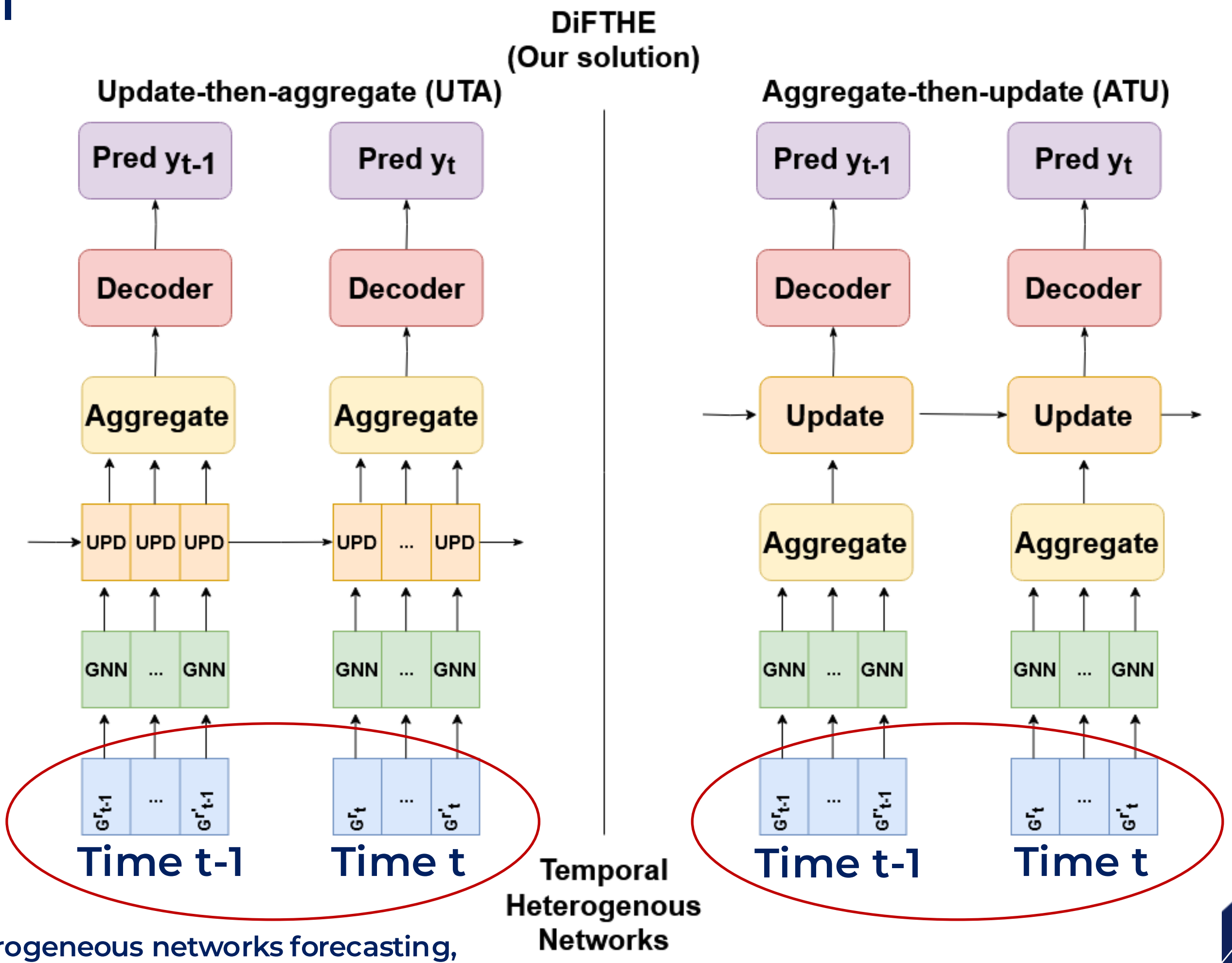
Decoder: solves a downstream task (e.g. link prediction, node classification)

Semantic aggregation: aggregates the embeddings over different relation types

Temporal update: Receives in input past and current node embeddings and produce new current node embeddings

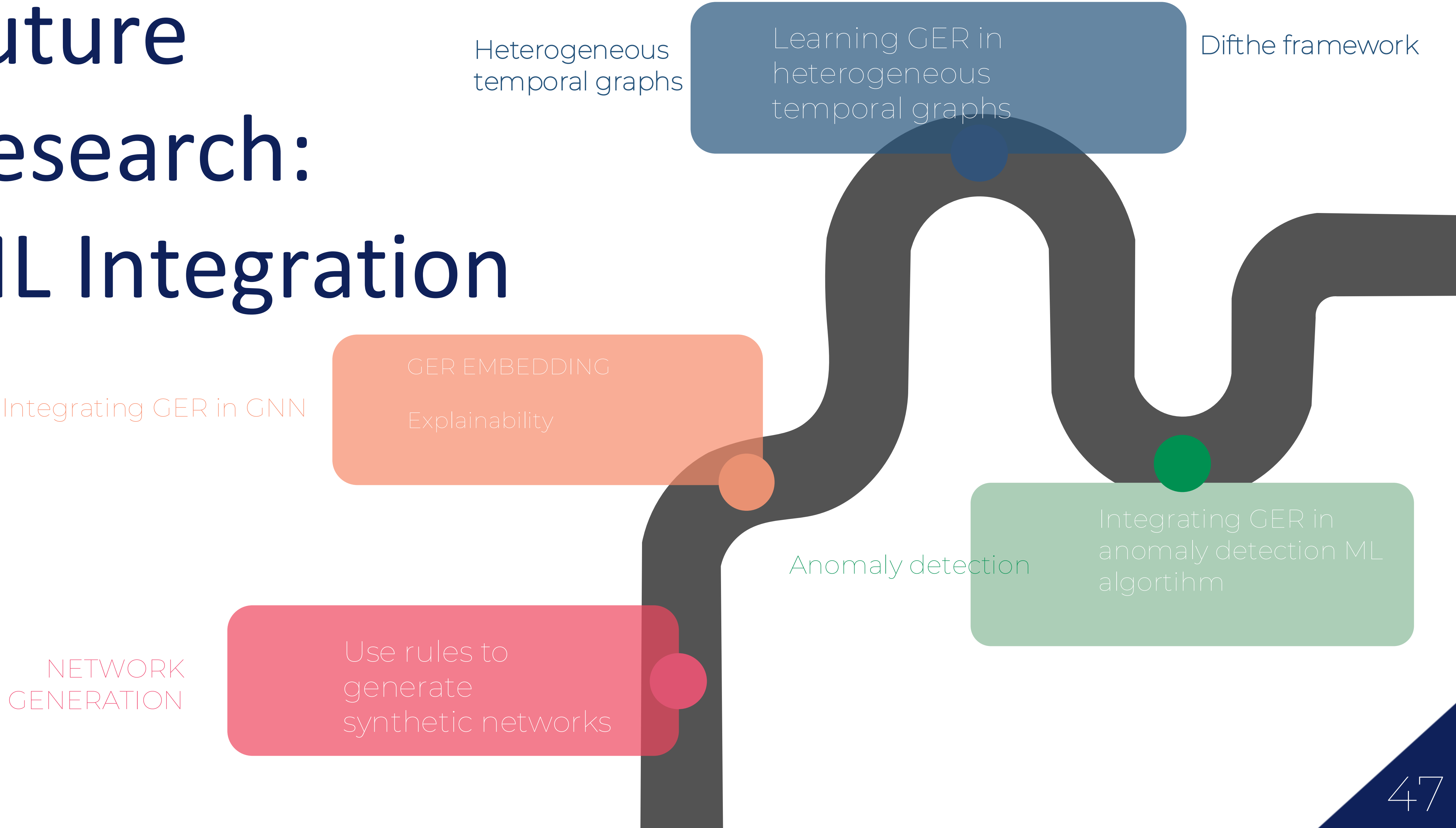
Topology: A GNN for each relation type

Input: heterogeneous graph snapshots



Dileo et al. (2024), A discrete-time deep learning framework for temporal heterogeneous networks forecasting, Proceedings of the 12th IEEE International Conference on Data Science and Advanced Analytics (DSAA)

Future Research: ML Integration



Some recent initiatives

On

Temporal/Evolutionary

Networks

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DI MILANO

TENET

Temporal Networks

Submissions deadline:
2nd March

Satellite in conjunction with

International School and Conference on Network Science 2026

June 1-5, 2026

Boston (US)

CCS25 https://sites.google.com/view/tenet-ccs25/home

-Netsci25 https://netsci2025.github.io

NetSci Boston 2026

Consider submitting your work here!

Springer

Springer.com

Applied Network Science

Call for Papers

Evolution of Networks

The study of network evolution represents a critical frontier in understanding complex systems across diverse domains, from social networks and biological systems to financial and transport networks. As networks constantly adapt and transform over time, understanding their evolutionary patterns and underlying mechanisms has become increasingly crucial for both theoretical advancement and practical applications. This topical collection seeks contributions that explore the dynamic nature of networks from multiple perspectives. We encourage interdisciplinary approaches that bridge multiple domains or methodologies, as well as both theoretical contributions and applied research that advance our understanding of network evolution. The peer-review process will begin as soon as submissions are received, not after the submission deadline, ensuring timely feedback.

NETS-AI
The convergence of Networks Science and AI

HONAI 2025: Higher-Order Networks meets AI @ NetSci25 //

https://hons-web.github.io/online/index.html //

Network Science meets AI
Special session of the 33th European Symposium on Artificial Neural Networks, Computational Intelligence and Machine Learning (ESANN)
https://sites.google.com/view/esann-netsai/home

Special issue on Applied Network Science: **Bridging Network Science and AI**

https://link.springer.com/collections/hafcebgfci

THANKS FOR YOUR ATTENTION

Finanziato dall'Unione europea NextGenerationEU

Ministero dell'Università e della Ricerca

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